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Code No. 16432/CORE

FACULTY OF SCIENCE
M.Sc. III Semester Examination, March 2021

Subject: Mathematics
Paper - I : Functional Analysis

Time: 2 Hours **Max. Marks: 80**

PART – A

Note: Answer any five questions. **(5 x 7 = 35 Marks)**

1 State and prove translation invariance lemma
2 Let T be a linear operator on a normed space X. Prove that T is continuous if and only if bounded.
3 State and prove Schwarz inequality.
4 State and prove direct sum theorem of two subspaces.
5 Let T be a bounded linear operator on a complex inner product space X and $<Tx, x> = 0$ for all $x \varepsilon X$, then prove that T = O.
6 Prove that the product of two bounded self-adjoint linear operators A and B on Hilbert space H is self-adjoint if and only if AB=BA.
7 Let X, Y be normed spaces and S,T ε B(X,Y). Then prove that
(i) $(S+T)^{x} = S^{x} + T^{x}$ (ii) $(\propto T)^{x} = \propto T^{x}$ for all scalars $\propto$.
8 Let X and Y be a normed spaces. Prove that the vector space XxY is a normed space under the norm $\|(x, y)\| = \max[\|x\|, \|y\|]$.

PART – B

Note: Answer any three questions. **(3 x 15 = 45 Marks)**

9 Let T be a linear operator. Prove that (i) The range R(T) is a vector space (ii) If dim D(T) = n < ∞, then dim R(T) ≤n. (iii) The null space N(T) is a vector space.

10 Prove that the space l^p, $(1 \leq p < \infty)$ is a Banach space under the norm $\|x\| = \left(\sum_{i=1}^{\infty} |x_i|^p\right)^{1/p}$.

11 Prove that the dual space of R^n is isomorphic with R^n.

12 State and prove Bessel's inequality.

13 State and prove Riesz's theorem for sesqualinear form.

14 Prove that the Hilbrt-adjoint operator $T*$ of T exists, is unique and is a bounded linear operator with norm $\|T*\| = \|T\|$.

15 State and prove open mapping lemma.

16 State and prove closed graph theorem.

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Code No. 16433/CORE

FACULTY OF SCIENCE
M.Sc. III Semester Examination, March 2021

Subject: Mathematics
Paper - II : General Measure & Integration

Time: 2 Hours Max. Marks: 80

PART – A

Note: Answer any five questions. **(5 x 7 = 35 Marks)**

1 Define a σ-finite measure and furnish an example of the same.
2 State and prove monotone convergence theorem with respect to a general measure.
3 Give an example to show that Hahn decomposition need not be unique.
4 Show that if ν_1 and ν_2 are singular measures with respect to the measure μ , then so is $\nu_1 + \nu_2$.
5 State the Caratheodory extension theorem.
6 Define a measure μ on an algebra $\mathcal{A}$ and define the outer measure μ^* induced by μ .
7 Under usual notations prove that if $A \in \mathcal{A}$, then $\mu(A) = \mu_*(A \cap E) + \mu^*(A \cap \tilde{E})$.
8 Prove that under usual notations if $E \subset F$, then $\mu_*(E) \leq \mu_*(F)$..

PART – B

Note: Answer any three questions. **(3 x 15 = 45 Marks)**

9 State and prove generalized lebesgue convergence theorem.

10 State and prove generalized Fatou's lemma.

11 Sate Radon-Nikodym theorem and sketch the proof of the same.

12 State and prove the Hahn's decomposition theorem.

13 State and prove Fubini's theorem.

14 State and prove Tonelli's theorem.

15 Under usual notations prove that if μ^* is a Caratheodory outer measure with respect to Γ , then every function in Γ is μ^* measurable.

16 Let μ be a measure on algebra $\mathcal{A}$ of subsets of X and E any subset of X. If B is the algebra generated by $\mathcal{A}$ and E and if $\tilde{\mu}$ is any extension of μ to B, then prove that μ^* $\mu^*(E) \geq \tilde{\mu}(E) \geq \mu_*(E)$.

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Code No. 16434 / Core

FACULTY OF SCIENCE

M.Sc. III-Semester Examination, March/April 2021

Sub: Mathematics / Applied Mathematics

Paper – III : Linear Algebra

Time: 2 Hours | Max.Marks:80

PART – A

Answer any five questions. **(5x7=35 Marks)**

1 Let $T:V\to V$ be a linear operator and $T\alpha = c\alpha$ for some scalar $c\in F$. If f is any polynomial then show that $f(T)\alpha = f(c)\alpha$.

2 Let $T:V\to V$ be a linear operator where V is a finite dimensional vector space. Then show that $N(T)$, the null space of T is invariant under T.

3 Let E be a projection and $f(x) = 13x^3 + 9x^2 - 12x + 15$ then evaluate $f(E)$.

4 Define cyclic subspace. With usual notation show that $Z(\alpha;T)$ is one dimensional if and only if α is a characteristic vector.

5 Find the rational canonical form of the matrix $A = J_4(\lambda)$ where $J_4(\lambda)$ is the 4 x 4 Jordan block with λ as the characteristic value.

6 Let T be a linear operator on a finite dimensional vector space over on algebraically closed filed. Then show that T is semi simple if T is a diagonalizable operator.

7 Let f be the a bilinear form on $\mathbf{R}^2$ defined by $f((x_1,y_1),(x_2,y_2)) = x_1y_1 + x_1y_2 + x_2y_1 + x_2y_2$. Then find the matrix of f in the ordered basis $\mathcal{B} = \{(2,4),(5,7)\}$

8 Let f be a bilinear form on a vector space V. Then show that f is symmetric if and only if its matrix A in some ordered basis is symmetric that is , $A^t = A$.

PART – B

Answer any three questions. **(3x15=45 Marks)**

9 Show that the matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$ is diagonalizable.

10 (i) Let V be the vector space of n x n matrices over the field F. Let A be a fixed n x n matrix. If the linear operator T : V → V is defined by T(B) = AB for any B ∈ V, then show that the minimal polynomial for T is the minimal polynomial for A.

(ii) Find the minimal polynomial of the matrix.

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

11 Let $V = W_1 \oplus W_2 \oplus \ldots\ldots \oplus W_k$. Then show that there exists k linear operators $E_1, E_2, \ldots. E_k$ such that (i) $E_i^2 = E_i$ (ii) $E_i E_j = 0$ if $i \neq j$ (iii) $I = E_1 + E_2 + \ldots. + E_k$ and (iv) the range of E_i is W_i. (Here $1 \leq i \leq k$))

12 Let $T : V \to V$ be a linear operator on a finite dimensional vector space over the field F. Suppose that the minimal polynomial for T decomposes over F into a product of linear polynomials. Then show that $T = D + N$ where D is a diagonalizable operator and N is a nilpotent operator. Further prove that the operators D, N are unique.

13 Let $T : V \to V$ be a linear operator and $f(x) \in F[x]$. If V_1, V_2 are any two T-invariant subspaces of V such that $V = V_1 \oplus V_2$ then show that $fV = fV_1 \oplus fV_2$.

14 Let N be a nilpotent operator on a finite dimensional vector V over F. Then show that there is a positive integer r such that the dimension of the null space of N is r. Also prove that there exists r positive integers $k_1, k_2, \ldots k_r$ such that $k_1 + k_2 + \ldots.. + k_r = n$ where $k_1 \geq k_2 \geq k_3 \ldots \geq k_r$.

15 Let f be a bilinear form on a finite dimensional vector space V. Let L_f and R_f be the linear transformations from V into V^* defined by $(L_f \alpha)(\beta) = f(\alpha, \beta) = (R_f \beta)(\alpha)$ for any $\alpha, \beta \in V$. Then show that rank (L_f) = rank (R_f).

16 (i) Define skew symmetric bilinear form. If f is a skew symmetric bilinear form on a vector space V show that $f(\alpha, \alpha) = 0$ for all $\alpha \in V$

(ii) Find the block diagonal representation of the bilinear form f defined on $\mathbb{R}^3$ given by $f((x_1, x_2, x_3), (y_1, y_2, y_3)) = x_1 y_2 - x_2 y_1 + 2x_1 y_3 - 2x_3 y_1 + x_3 y_2 - x_2 y_3$. Also find a basis of $\mathbb{R}^3$.

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Code No. 16435/CORE/A

FACULTY OF SCIENCE
M.Sc. III Semester Examination, July 2021

Subject: Mathematics / Applied Mathematics / Maths with Computer Science
Paper - IV : Operations Research

Time: 2 Hours **Max. Marks: 80**

PART – A

Note: Answer any five questions. **(5 x 7 = 35 Marks)**

1 Solve the following LPP graphically:
Minimize $z = 5x_1 + 4x_2$ subject to the constraints $x_1 - 2x_2 \le 1$, $x_1 + 2x_2 \ge 3$; $x_1, x_2 \ge 0$.

2 Convert the LPP: Maximize $z = 3x_1 + 2x_2$ subject to the constraints $4x_1 - x_2 \le 5$, $x_1 + 2x_2 = 7$; x_1 is unrestricted in sign and $x_2 \ge 0$, into dual.

3 Explain degeneracy in transportation problems.

4 State and prove reduction theorem.

5 What is dynamic programming and what sort of problems can be solved by it?

6 Write essential characteristics of dynamic programming problems.

7 Explain various basic steps in PERT/CPM techniques.

8 Write rules for drawing network diagrams.

PART – B

Note: Answer any three questions. **(3 x 15 = 45 Marks)**

9 Solve the following LPP:
Maximize $z = 5x_1 + 3x_2$
subject to the constraints $x_1 + x_2 \le 2$, $5x_1 + 2x_2 \le 10$, $3x_1 + 8x_2 \le 12$; $x_1, x_2 \ge 0$.

10 Using two-phase simplex method, solve the LPP: Minimize $z = \frac{15}{2}x_1 - 3x_2$ subject to the constraints $3x_1 - x_2 - x_3 \ge 3$, $x_1 - x_2 + x_3 \ge 2$; $x_1, x_2, x_3 \ge 0$.

11 Find an initial basic feasible solution to the following transportation problem using lowest cost entry method and Vogel's approximation method.

	D1	D2	D3	D4	Supply
S1	23	27	16	18	30
S2	12	17	20	51	40
S3	22	28	12	32	53
Demand	22	35	25	41	123

12 Solve the following travelling salesman problem.

To

	A	B	C	D	E
A	∞	4	7	3	4
B	4	∞	6	3	4
C	7	6	∞	7	5
D	3	3	7	∞	7
E	4	4	5	7	∞

From

..2

13 Using dynamic programming, divide a positive number c into n parts such that their product is maximum.

14 If $p_1 + p_2 + p_3 + \ldots\ldots p_n = 1$, show that the sum $z = p_1 \log p_1 + p_2 \log p_2 + p_3 \log p_3 + \ldots\ldots + p_n \log p_n, p_i \geq 0$ for all i, is minimum when $p_1 = p_2 = p_3 = \ldots\ldots = p_n = \frac{1}{n}$.

15 A project consists of a series of tasks labelled A,B,C,D,E,F,G,H,I with the following relationships:
W <X,Y means X and Y cannot start until W is completed; X,Y < W means W cannot start until both X and Y are completed.
With this notation, construct the network diagram having the following constraints:
A < D,E ; B,D<F; C<G ; B,G< H; F,G<I
Find also the critical path and project completion time, when the time (in days) of completion of each task is as follows:

Task	A	B	C	D	E	F	G	H	I
Time	23	8	20	16	24	18	19	4	10

16 A project consists of the following activities and time estimates.

Activity	Time estimates (in weeks)		
	Optimistic	Most likely	Pessimistic
1-2	1	1	7
1-3	1	4	7
1-4	2	2	8
2-5	1	1	1
3-5	2	5	14
4-6	2	5	8
5-6	3	6	15

(i) Draw the network diagram for the project.
(ii) Determine the expected time and variance of each activity.
(iii) Determine the critical path and the expected project completion time.

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Code No. 16439/CORE

FACULTY OF SCIENCE

M.Sc. III Semester Examination, July 2021

Subject: Mathematics / Applied Mathematics

Paper – V (B): Numerical Analysis

Time: 2 Hours **Max. Marks: 80**

PART – A

Note: Answer any five questions. **(5 x 7 = 35 Marks)**

1 Perform five iterations of the bisection method to obtain the smallest positive root of the equation $x^3 - 5x + 1 = 0$.

2 Apply Newton-Raphson method to determine a root of the equation $f(x) = \cos x - xe^{x} = 0$.

3 Fine the inverse of the coefficient matrix of the system $\begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 6 \\ 4 \end{bmatrix}$ by the Gauss-Jordan method and hence solve the system.

4 Explain SOR method to solve the system of equations $Ax = b$, where A is the coefficient matrix.

5 Derive the Newton's divided difference interpolating polynomial for the data

x_0	x_1	x_2	x_n
$f(x_0)$	$f(x_1)$	$f(x_2)$	$f(x_n)$.

6 Obtain the least square polynomial approximation of degree two for $f(x) = x^{1/2}$ *on* $[0,1]$.

7 Evaluate the integral $I = \int_0^1 \frac{dx}{1+x}$ using Gauss-Legendre three point formula.

8 Use the Euler's method to solve numerically the initial value problem $u' = 2tu^2, u(0) = 1$ *with* $h = 0.2$.

PART – B

Note: Answer any three questions. **(3 x 15 = 45 Marks)**

9 Define rate of convergence. Determine the rate of convergence of secant method.

10 Perform five iterations of the Muller's method to find the root of the equation $f(x) = \cos_{n} - xe^{x} = 0$. Use the initial approximations $x_0 = -1, \ x_1 = 0, \ x_2 = 1$.

11 Determine the inverse of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix}$ using the partition method.

12 Perform four interations of the Gauss-seidel iterative method to solve the system of equations

$$2x_1 - x_2 = 7$$
$$-x_1 + 2x_2 - x_3 = 1$$
$$-x_2 + 2x_3 = 1.$$

13 For the following data;

x	$f(x)$	$f'(x)$
−1	1	-5
0	1	1
1	3	7

estimate the values of $f(-0.5)$ and $f(0.5)$ using the Hermite interpolation.

14 Find the least squares approximation of second degree for the descrete data

x :	-2	-1	0	1	2
$f(x)$:	15	1	1	3	19

15 Derive Simpson's rule and hence show that the error of approximation in the Simpson's rule is $R_2 = -\frac{(b-a)^5}{2880} f^{iv}(x)$ in the interval $[a,b]$.

16 Use the Runge-Kutta fourth order method to solve the initial value problem $u' = -2tu^2$, $u(0) = 1$, with $h = 0.2$ on the interval $[0,1]$.

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